

Complements about the infinitude of composite numbers

Dimitri Karpov

ABSTRACT

The author recalls a few results in algebraic humor theory, and corrects a mistake of Bruno Winckler in [Win] about the infinitude of composite numbers.

Algebraic humor theory mainly deals with the use of algebra in mathematical humor. To justify the existence of this theory, we have the fundamental theorem of algebraic humor theory:

Theorem 1 (Fundamental theorem of algebraic humor theory): Every nonnegative integer is of interest.

Thus, since they are of interest, we can study to what extent they are of humoristic interest.

Proof. Suppose no one gives a single f- about some nonnegative integers. Let X be the set of these integers; since $X \subseteq \mathbb{N}$, there exists a smallest integer in X , that I denote by n . Well, this property of n is quite interesting! This leads to a contradiction. \square

One can prove that the same holds for a noetherian ring: every ideal is of interest (the case of the ring \mathbb{Z} gives our theorem).

Examples. 2 is the oddest prime; 14 is the smallest integer such that $\varphi(n) = 14$ has no solution, where φ is the totient function; 23 is one of the only two prime numbers that cannot be written as a sum of eight cubic integers; the 43th Gödel number is the first one that is not an integer, 1729 is the smallest number expressible as the sum of two positive cubes in two different ways, 105 is the smallest number expressible as a produit of 35 and 3 in two different ways, *etc.*

Among the integers, the prime numbers are the most important, since they produce a large class of jokes. For instance, I think everyone knows the jokes “proving” that every odd number greater than 3 is a prime number. For a long time, humor theorists studied the prime numbers, hoping that every integer is a prime number. Unfortunately, the situation is not as funny as it should:

Definition 1: A composite number is an integer greater than 2 that is not a prime number.

Proposition 1: There is an infinity of composite numbers.

Proof. First, we need a lemma:

Lemma 1: There exists at least one composite number.

Proof of the lemma. Indeed, let suppose there is no composite number. Then, using the following identity, certainly proved in [Bou]:

$$\prod_{x \in \emptyset} x = 1,$$

multiply them all and you will get one. \square

Now we can prove the proposition, following the idea of [Win], Théorème 13, page 20 (the reader will notice that the author of [Win] forgot to prove the existence of composite numbers). Suppose that the set of composite numbers is finite. Then, multiply them together, and you will get new ones outside of this set. \square

You can imagine how many unfunny jokes there are, because of these composite numbers. For example, the jokes linking a given number x^2 with a geometric square are among the worst jokes ever, you do not want to know any of them. But a few authors use an ingenious descent method to reduce composite numbers' jokes to prime numbers' jokes.

Example 1. A pair of discrete mathematicians are in a field. One asks: "I wonder how many sheeps there are in this field?"

The second replies, "There are 200 sheeps."

The first is astonished, "How did you count that so fast?"

"Simple, I counted the legs and divided by four!"

An algebraist who happens to be passing by at the time scoffs, "Applied mathematicians...", shaking his head in disgust. "A finite field can't be of order 200. Surely you've miscounted 199..."

Example 2. The mathematician Euler wrote the celebrated " $2 + 2 = 5$ " joke thanks to a transfer principle, that functorially transforms the composite number 4 into the prime number 5, so that the identity " $2 + 2 = 5$ " only involves prime numbers; for more details, see [Win], chapitre 7, page 199. Strangely enough, 4 is the core of several mathematical jokes, and the understanding of its fun potential is an open problem in mathematics.

References

[Bou] Bourbaki Nicolas, *Éléments de mathématiques* x , $x \in \{1, 2\}$, Springer, 2006.

[Win] Winckler Bruno, *Blagues mathématiques et autres curiosités*, Ellipses, 2011.